(12)

AD

BRL

BRL R 1937

AD A030703

REPORT NO. 1937

TRANSIENT TEMPERATURES PRODUCED IN SOLID CYLINDERS BY A NUCLEAR THERMAL PULSE

Ennis F. Quigley

September 1976

Approved for public release; distribution unlimited.

USA BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
BRL Report No. 1937

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
(6) TRANSIENT TEMPERATURES PRODUCED IN SOLID CYLINDERS BY A NUCLEAR THERMAL PULSE.

5. TYPE OF REPORT & PERIOD COVERED
(9) FINAL rept.,

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
(10) Ennis F. Quigley

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
USA Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland 21005

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
1W162118AH75

11. CONTROLLING OFFICE NAME AND ADDRESS
US Army Materiel Development and Readiness Command
5001 Eisenhower Avenue
Alexandria, Virginia 22314

12. REPORT DATE
(11) Sep[redacted]76 (12)

13. NUMBER OF PAGES
23 20p

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
(14) BRL-1937

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.
(16) DA-1-W-162118-AH-75

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DDC OCT 13 1976

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Nuclear Thermal Environment
Transient Temperatures
Solid Cylinder

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (DTSebold)

The analytical expression for the transient temperature field in an isotropic, homogeneous, finite length, solid cylinder whose lateral surface is subjected to heating by a nuclear thermal radiation environment is derived. This expression provides a convenient means for the nondimensional representation and parametric analysis of the temperature field in the cylinder. In addition, this temperature equation can be used in analytical analysis of those effects dependent on temperature or temperature change, e.g., thermal stresses in cylinders.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

050 450

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

1B

TABLE OF CONTENTS

		Page
I.	INTRODUCTION	5
II.	TEMPERATURE EQUATIONS	7
III.	NUMERICAL RESULTS	15
IV.	CONCLUSION	16
	REFERENCES	20
	GLOSSARY OF TERMS	21
	DISTRIBUTION LIST	23

I. INTRODUCTION

The atmospheric detonation of a nuclear weapon produces a high temperature air plasma which is commonly referred to as a "fireball". This fireball begins to emit thermal radiation, mostly in the visible and infrared regions of the spectrum, into the surrounding atmosphere immediately after it is formed. Although the fireball has a finite diameter, it is considered to be a point source for calculational purposes, and the thermal radiation environment produced is expressed in terms of a time dependent thermal irradiance. The general characteristics of this irradiance at any point outside the fireball are shown by the curve in Figure 1. This curve is generally approximated by

$$H(t) = H_o \left[\frac{2.2\left(\frac{t}{t_o}\right)^2}{1 + 1.69\left(\frac{t}{t_o}\right)^{3.6}} + \frac{0.206\, e^{-3.6\left(\frac{t}{t_o} - 1.18\right)^2}}{1 + \left(\frac{t}{1.6t_o}\right)^{10}} \right] \quad (1)$$

where t_o is a function of the weapon yield and H_o is a function of weapon yield, distance from point of weapon detonation, and transmittance of the atmosphere. Table 1 contains values of t_o, H_o and $\int_o^\infty H(t)\,dt$ for several weapon yields and distances. In theory the thermal environment is of infinite duration.

TABLE I. Nuclear Pulse Parameters

Yield (Kt)	Distance (km)	t_o (s)	Ho (MWm^{-2})	$\int_o^\infty Hdt$ (MJm^{-2})
1	0.32	0.04	11.32	1.25
10	0.70	0.11	11.26	3.26
100	1.51	0.31	8.96	7.31
1000	3.26	0.87	4.65	14.08

However, for practical applications the time duration of the environment is taken to be $10t_o$ since $\int_o^{10t_o} H(t)dt = 0.85 \int_o^\infty H(t)dt$ and $H(t) < 0.03\, H_o$ for $t > 10t_o$.

The effects of the thermal environment on exposed materials are due to the absorption of all or part of the radiant energy incident on the exposed surfaces of the materials. The absorption of this radiant energy by the lateral surface of a cylinder will result in a transient temperature rise, a knowledge of which is essential for predicting the response of the cylinder for those effects which are a function of temperature or temperature change. Because of the transient heating, a numerical

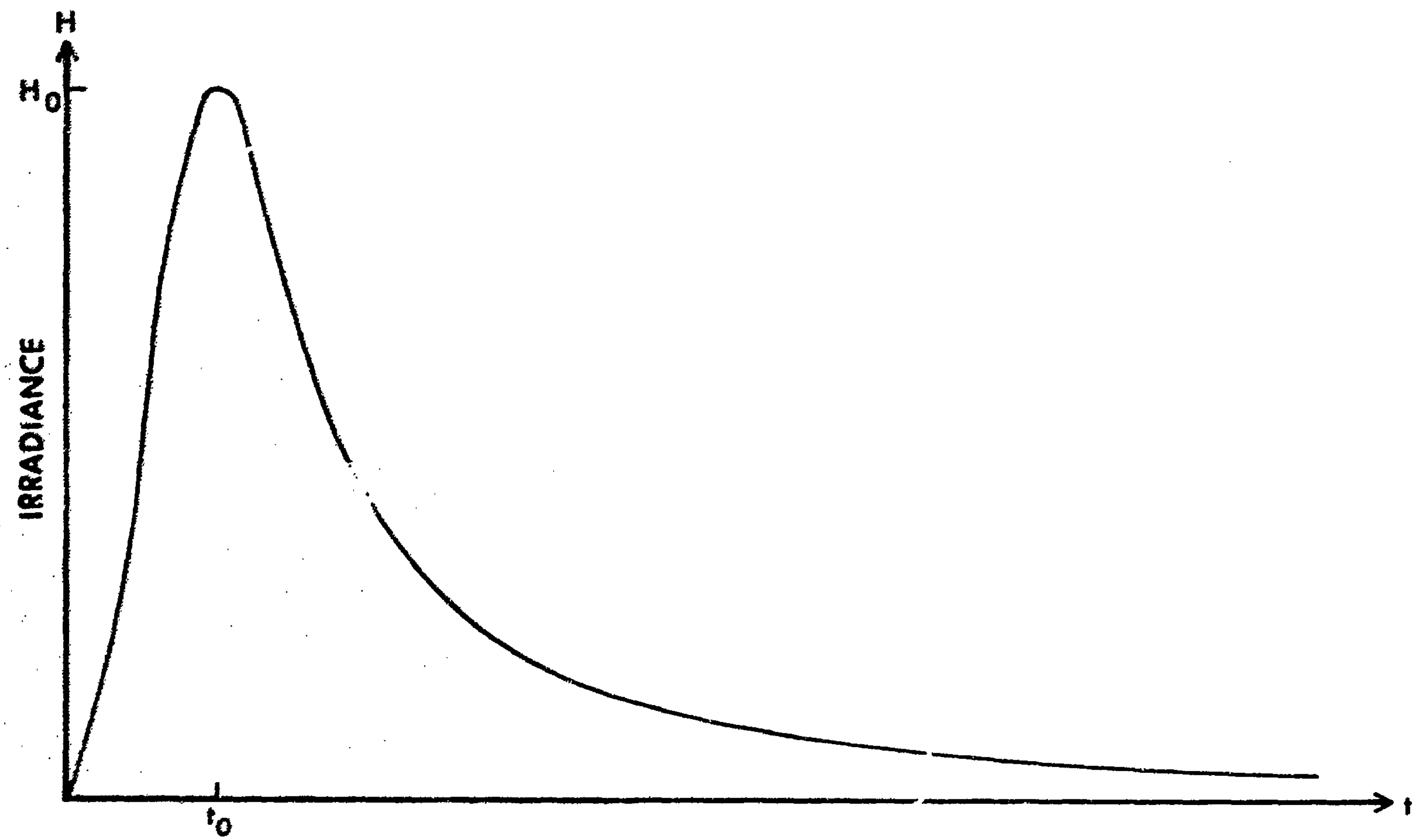

Figure 1. Nuclear Thermal Pulse

calculation of the temperature using finite difference or finite element methods is generally employed. Although these methods are of great value for solving a specific problem, they are not convenient as analytical methods for nondimensional representation or parametric analysis of the temperature field in the cylinder.

This report describes a deviation of the transient temperature field in an isotropic, homogeneous, finite length, solid cylinder of radius r_o whose lateral surface is subjected to heating by a nuclear thermal radiation environment. Ojalvo's modified separation-of-variables method[1] is used to solve the transient heat conduction equation under the following assumptions:

1. the thermal properties of the cylinder are independent of temperature,
2. the thermal radiation is absorbed at the surface,
3. convection and radiation heat losses by the cylinder can be neglected,
4. the transient temperature field in the cylinder is independent of the axial coordinate, and
5. the initial temperature field in the cylinder is uniform.

II. TEMPERATURE EQUATIONS

The thermal irradiance at the lateral surface of the cylinder is expressed by

$$H(\theta,t) = \begin{cases} H_o f(t) \cos\theta & 0 \le \theta \le \frac{\Pi}{2} \\ 0 & \frac{\Pi}{2} \le \theta \le \frac{3\Pi}{2} \\ H_o f(t) \cos\theta & \frac{3\Pi}{2} \le \theta \le 2\Pi \end{cases} \tag{2}$$

where f(t) is the time dependent portion of (1). The transient temperature field in the cylinder is governed by the following equations

$$\nabla^2 T(r,\theta,t) = \frac{c\rho}{\kappa} \frac{\partial T(r,\theta t)}{\partial t} \tag{3}$$

where

$$\nabla^2 \equiv \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r}\right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} , \tag{4}$$

---

[1] *J.U. Ojalvo, "Conduction with Time-Dependent Heat Sources and Boundary Conditions," International Journal of Heat and Mass Transfer, Vol 5, 1962, pp. 1105-1109.*

$$\frac{\partial T}{\partial r} = \begin{cases} \dfrac{H_o f \cos\theta}{\kappa} & 0 \le \theta \le \dfrac{\Pi}{2} \\ 0 & \dfrac{\Pi}{2} \le \theta \le \dfrac{3\Pi}{2} \\ \dfrac{H_o f \cos\theta}{\kappa} & \dfrac{3\Pi}{2} \le \theta \le 2\Pi \end{cases} \tag{5}$$

at $r = r_o$,

$$\frac{1}{r}\frac{\partial T}{\partial \theta} = 0, \tag{6}$$

at $\theta = 0, \Pi$ (because of symmetry), and

$$T(r,\theta,0) = 0^{*}. \tag{7}$$

The modified separation-of-variables method assumes that the solution of (3) can be expressed as

$$T = \sum_m \sum_n \Psi_{mn}(t)\,\Phi_{mn}(r,\theta) + T_o(r,\theta) f(t). \tag{8}$$

By substituting (8) into (3) one has

$$\sum\sum \Psi_{mn}\nabla^2\Phi_{mn} + \nabla^2 T_o f = \frac{c\rho}{\kappa}\left[\sum\sum \dot{\Psi}_{mn}\Phi_{mn} + T_o\dot{f}\right]. \tag{9}$$

The method further assumes that

$$T_o = \sum a_{mn}\Phi_{mn} \tag{10}$$

and that either

$$\nabla T_o^{\,2} = 0 \tag{11}$$

or

$$\nabla^2 T_o = \sum b_{mn}\Phi_{mn} \tag{12}$$

---

* *No generality is lost by assuming $T = 0$ since the temperature of the cylinder can be obtained by adding the initial constant temperature to T and only the temperature difference is required for the stress and displacement fields.*

The substitution of (10) and (12)* into (9) yields

$$\sum\sum \Psi_{mn} \nabla^2 \Phi_{mn} = \frac{c\rho}{\kappa} \sum\sum \left[\dot{\Psi}_{mn} + a_{mn}\dot{f} - \frac{\kappa}{c\rho} b_{mn} f \right] \Phi_{mn} \quad (13)$$

If (13) is equated termwise and divided by $\Psi_{mn} \Phi_{mn}$ and if $-\lambda_{mn}^2$'s are chosen for the separation constants, one obtains the following differential equations:

$$\nabla^2 \Phi_{mn} + \lambda^2_{mn} \Phi_{mn} = 0 \quad (14)$$

and

$$\dot{\Psi}_{mn} + \lambda^2_{mn} \frac{\kappa}{c\rho} \Psi_{mn} = -a_{mn}\dot{f} + \frac{\kappa}{c\rho} b_{mn} f \quad (15)$$

The solution of (14) and (15) along with T_0 when substituted into (8) provides the transient temperature field in the cylinder.

The boundary and initial conditions for Φ_{mn}, T_0, and Ψ_{mn} are obtained by substituting (8) into (5), (6), and (7). This substitution yields

$$\sum\sum \Psi_{mn} \frac{\partial \Phi_{mn}}{\partial r} = \begin{cases} \left[\frac{H_0}{\kappa} \cos\theta - \frac{\partial T_0}{\partial r} \right] f & 0 \le \theta \le \frac{\pi}{2} \\ -\frac{\partial T_0}{\partial r} f & \frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \\ \left[\frac{H_0}{\kappa} \cos\theta - \frac{\partial T_0}{\partial r} \right] f & \frac{3\pi}{2} \le \theta \le 2\pi \end{cases} \quad (16)$$

at $r = r_0$,

$$\sum\sum \Psi_{mn} \frac{\partial \Phi_{mn}}{r\partial\theta} = -\frac{\partial T_0}{r\partial\theta} f \quad (17)$$

at $\theta = 0, \pi$, and

$$\sum\sum \Psi_{mn} \Phi_{mn} = -T_0 f \quad (18)$$

---

* *The use of (12) is more general than the use of (11) since (11) is a special case of (12) in which all the* b_{mn}*'s are zero.*

at t = 0. From (16), (17), (18) and Figure 1 the following boundary and initial conditions for Φ_{mn}, T_o, and Ψ_{mn} are deduced:

$$\frac{\partial \Phi_{mn}}{\partial r} = 0^* \tag{19}$$

at $r = r_o$,

$$\frac{\partial \Phi_{mn}}{r \partial \theta} = 0^* \tag{20}$$

at $= 0, \theta, \Pi$

$$\frac{\partial T_o}{\partial r} \begin{cases} \frac{H_o}{\kappa} \cos \theta & 0 \leq \theta \leq \frac{\Pi}{2} \\ 0 & \frac{\Pi}{2} \leq \theta \leq \frac{3\Pi}{2} \\ \frac{H_o}{\kappa} \cos \theta & \frac{3\Pi}{2} \leq \theta \leq 2\Pi \end{cases} \tag{21}$$

at $r = r_o$,

$$\frac{\partial T_o}{r \partial \partial \theta} = \theta \tag{22}$$

at $\theta = 0, \Pi$, and

$$\Psi_{mn} = 0 \tag{23}$$

at t = 0.

Equation (14) is a two dimensional Helmholtz's differential equation, the general solutions of which are of the form

$$\Phi_{on} = J_o (\lambda_{on} r) (A_{on} + B_{on} \theta) \tag{24a}$$

for m = 0 and

$$\Phi_{mn} = (A_{mn} J_m(\lambda_{mn} r) + B_{mn} Y(\lambda_{mn} r))(C_m \cos m\theta + D_m \sin m\theta) \tag{24b}$$

for m>0.

In order for these solutions to be well behaved at r = 0, the B_{mn}'s must be equal to zero. Also from (20) it is readily seen that the B_{on}'s and the D_m's must also equal zero and that m = 0, 1, 2, 3, ... Equation (24) can now be rewritten as

$$\Phi_{mn} = A_{mn} J_m(\lambda_{mn} r) \cos m\theta \tag{25}$$

* *This equation insures a homogeneous boundary condition for (14).*

The substitution of (25) into (19) gives

$$J_m{}' \, (\lambda_{mn} r_o) = 0 \tag{26a}$$

or

$$(\lambda_{mn} r_o) \frac{J_{m+1} (\lambda_{mn} r_o)}{J_m \, (\lambda_{mn} r_o)} = m \tag{26b}$$

From (26a) one sees that the $\lambda_{mn} r_o$'s are the n-th positive roots of (26a).

Equation (11) cannot be used in determining T_o. The solution of this two dimensional Laplace's differential equation which is well behaved at $r = 0$ and satisfies (22) is

$$T_o = C_o + \sum_{m=1}^{\infty} C_m r^m \cos n \, \theta \tag{27}$$

the substitution of (27) into (21) yields

$$\sum_{m=1} m C_m r_o^{m-1} \cos m \, \theta = \begin{cases} \frac{H}{\kappa} \cos \theta & 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \\ \frac{H}{\kappa} \cos \theta & \frac{3\pi}{2} \leq \theta \leq 2\pi \end{cases} \tag{28}$$

By expressing the right side of (28) in terms of the following Fourier cosine series:

$$\frac{H_o}{\kappa} \left[\frac{1}{\pi} + \frac{\cos \theta}{2} + \frac{2}{\pi} \sum_{n=2,4,6,\ldots}^{\infty} \frac{(-1)^{\frac{n}{2}+1}}{(n^2-1)} \cos n \, \theta \right], \tag{29}$$

one can rewrite (28) as

$$\sum_{m=1}^{\infty} m C_m r_o^{m-1} \cos m \, \theta = \frac{H_o}{\kappa} \left[\frac{1}{\pi} + \frac{\cos \theta}{2} + \frac{2}{\pi} \times \sum_{\substack{n=2 \\ \text{even}}}^{\infty} \frac{(-1)^{\frac{n}{2}+1}}{(n^2-1)} \cos n \, \theta \right] \tag{30}$$

It is readily seen from (30) that (27) cannot satisfy the boundary conditions at $r = r_o$ because of the first term of (27). This term represents the uniform heating of the entire lateral surface of the cylinder, and the form of T_o which is obtained by solving the uniform

heating case is

$$T_o = \frac{H_0}{2\Pi\kappa r_o} r^2 \tag{31}$$

Consequently, it is assumed that

$$T_o = \frac{H}{2\Pi\kappa r_o} r^2 + C_o + \sum_{m=1}^{\infty} C_m r^m \cos m\,\theta \tag{32}$$

where the values of the C_m's, except for C_o, are obtained by equating (30) termwise. From (30) one sees that

$$C_1 = \frac{H_0}{2\kappa} \tag{33}$$

and

$$C_m = -\frac{2H_0 \cos\left(\frac{m\Pi}{2}\right)}{\Pi m(m^2-1)\kappa r_o^{m-1}} \tag{34}$$

for $m > 1$.

In order to solve

$$\dot{\Psi}_{mn} + \frac{\kappa}{c\rho}\lambda^2_{mn}\Psi_{mn} = -a_{mn}\dot{f} + \frac{\kappa}{c\rho} b_{mn} f \tag{15}$$

the coefficients, a_{mn} and b_{mn}, must be evaluated. The a_{mn}'s can be evaluated from (10) and (32) and the b_{mn}'s from (12) and (32). From (10) and (32)

$$\frac{H_o}{2\Pi\kappa r_o} r^2 + C_o + \sum C_m r^m \cos m\,\theta = a_{oo} + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} a_{mn} J_m(\lambda_{mn} r) \cos m\,\theta \tag{35}$$

Equating (35) termwise one obtains

$$\frac{H_o}{2\Pi\kappa r_o} r^2 + C_o = a_{oo} + \sum_{n=1}^{\infty} a_{on} J_o(\lambda_{on} r) \tag{36}$$

and

$$C_m r^m = \sum_{n=1} a_{mn} J_m(\lambda_{mn} r). \tag{37}$$

Using the orthogonality properties of the Bessel functions, the a_{mn}'s can be evaluated from

$$a_{on} = \frac{\frac{H_o}{2\Pi\kappa r_o}\int_o^{r_o} r^3 J_o(\lambda_{on} r)dr + C_o\int_o^{r_o} rJ_o(\lambda_{on} r)dr}{\int_o^{r_o} rJ_o^2(\lambda_{on} r)dr} \tag{38}$$

and

$$a_{mn} = C_m \frac{\int_o r^{m+1} J_m(\lambda_{mn} r)dr}{\int_o rJ_m^2(\lambda_{mn} r)dr} . \tag{39}$$

The integration of (38) yields

$$a_{oo} = \frac{4H_o r_o}{\Pi\kappa} + C_o \tag{40a}$$

for $\lambda_{oo} = 0$ and

$$a_{on} = \frac{2H_o r_o}{\Pi\kappa(\lambda_{on} r_o)^2 J_o(\lambda_{on} r)} \tag{40b}$$

for $n = 1$. For the remaining a_{mn}'s the integration of (39) yields

$$a_{mn} = \frac{2C_m \lambda_{mn} r_o^{m+1} J_{m+1}(\lambda_{mn} r_o)}{\left[(\lambda_{mn} r_o)^2 - n^2\right] J_m^2(\lambda_{mn} r_o)} \tag{41}$$

By substituting (26b), (33) and (34) into (41) one obtains

$$a_{m1} = \frac{H_o(\lambda_{m1} r_o)^2}{\kappa\left[(\lambda_{m1} r_o)^2 - 1\right] J_1(\lambda_{m1} r_o)} \tag{42a}$$

for $m \geq 1$ and

$$a_{mn} = -\frac{2H_o r_o^2 \cos(m\Pi/2)}{\Pi\kappa\left[m^2 - 1\right]\left[(\lambda_{mn} r_o)^2 - m^2\right] J_m(\lambda_{mn} r_o)} \tag{42b}$$

for $m \geq 1$ and $n > 1$. From (12) and (32)

$$\frac{2H_o}{\Pi\kappa r_o} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} b_{mn} J_m(\lambda_{mn} r) \cos m\theta \tag{43}$$

Using the orthogonality properties of the Bessel functions and the cosine functions, the b_{mn}'s can be evaluated from

$$b_{mn} = \frac{2H_o}{\Pi\kappa r_o} \frac{\int_o^{r_o} rJ_m(\lambda_{mn}r)\,dr \int_o^{2\Pi} \cos m\theta d\theta}{\int_o^{r_o} rJ_m^2(\lambda_{mn}r_o)\,dr \int_o^{2\Pi} \cos^2 m\theta d\theta} \tag{44}$$

The integration of (44) yields

$$b_{oo} = \frac{2H_o}{\Pi\kappa r_o} \tag{45a}$$

and

$$b_{mn} = 0 \tag{45b}$$

for $m \geq 0$ and $n \geq 1$.

For $\lambda_{oo} = 0$ equation (15) reduces to

$$\dot{\Psi}_{oo} = -a_{oo}\dot{f} + \frac{\kappa}{c\rho} b_{oo} f, \tag{46}$$

the integration of which is

$$\Psi_{o1} = -a_{oo}f(t) + \frac{\kappa}{c\rho} b_{oo} \int_o^t f(t)dt + B_{oo} \tag{47}$$

From the initial condition equation (23), one sees that B_{oo} must equal zero. For the remaining λ_{mn}'s, the general solution of (15) can be obtained by using the integrating factor $e^{\lambda_{mn}^2 \frac{\kappa}{c\rho} t}$.

The use of this factor results in

$$\Psi_{mn} = -a_{mn}e^{-\lambda_{mn}^2 \frac{\kappa}{c\rho} t} \int_o^t e^{\lambda_{mn}^2 \frac{\kappa}{c\rho} t} f(t)dt + B_{mn}e^{-\lambda_{mn}^2 \frac{\kappa}{c\rho} t}. \tag{48}$$

One also sees from (23) that the B_{mn}'s must equal zero. The integration of the right hand side of (48) by parts yields

$$\Psi_{mn} = -a_{mn}f(t) + a_{mn}\lambda_{mn}^2 \frac{\kappa}{c\rho} e^{-\lambda_{mn}^2 \frac{\kappa}{c\rho} t} \times \int_o^t e^{+\lambda_{mn}^2 \frac{\kappa}{c\rho} t} f(t)dt \tag{49}$$

The substitution of (25), (32), (40), (42), (45), (47) and (49) into gives

$$T = \frac{2H_0}{\Pi\rho c r_0}\left\{\int_0^t f(t)dt + \sum_{n=1}^{\infty} \frac{e^{-\lambda_{on}^2 \frac{\kappa}{c\rho} t}}{J_0(\lambda_{on} r_0)}\left[\int_0^t e^{\lambda_{on}^2 \frac{\kappa}{c\rho} t}\right.\right.$$

$$\left. \times f(t)dt\right] J_0(\lambda_{on} r) + \frac{\Pi}{2}\sum_{n=1}^{\infty} \frac{[\lambda_{1n} r_0]^2 e^{-\lambda_{1n}^2 \frac{\kappa}{c\rho} t}}{[(\lambda_{1n} r_0)^2 - 1]\, J_1(\lambda_{1n} r_0)}$$

$$\times\left[\int_0^t e^{\lambda_{1n}^2 \frac{\kappa}{c\rho} t} f(t)dt\right] J_1(\lambda_{1n} r)\cos\theta - 2\sum_{m=2}^{\infty}\sum_{n=1}^{\infty} \frac{[\lambda_{mn} r_0]^2}{[m^2-1]}$$

$$\times \frac{\cos(m\Pi/2)\, e^{-\lambda_{mn}^2 \frac{\kappa}{c\rho} t}}{[(\lambda_{mn} r_0)^2 - m^2]\, J_m(\lambda_{mn} r_0)}\left[\int_0^t e^{\lambda_{mn}^2 \frac{\kappa}{c\rho} t} f(t)dt\right]$$

$$\left. \times J_m(\lambda_{mn} r)\cos m\theta \right\} \tag{50}$$

where f(t) is the time dependent portion of (1).

III. NUMERICAL RESULTS

For simplicity and generality, the derived temperature field equation will be numerically evaluated in terms of the following dimensionless quantities

$$r^* = \frac{r}{r_0} \qquad t^* = \frac{t}{t_0}$$
$$\beta^* = \frac{\kappa t_0}{c\rho r_0^2} \qquad T^* = \frac{T}{\frac{2H_0 t_0}{\Pi\rho c r_0}} \tag{51}$$

In addition, (1) is approximated by the following Fourier series:

$$f(t) = H_o \left[\frac{C_o}{2} + \sum_{l=1}^{\rho} C_l \cos \frac{\Pi l t}{5t_o} + d_l \sin \frac{l \Pi t}{5t_o} \right] \tag{52}$$

If (1) were used in evaluating (50), the integrals would have to be computed numerically. Since the use of (52) allowed for the direct integration of these integrals, such a representation is both logical and convenient, regardless of the number of terms required for accuracy. All of the calculations were carried out by using a UNIVAC 1108 digital computer and the number of terms used in evaluating the double series solution were such as to insure at least three digit convergence.

Figures 2, 3, and 4 are plots of the radial temperature distribution for various values of β^*, t^* and θ. A comparison of the temperatures calculated using (50) and those calculated using CINDA-3G[2], a finite differencing heat transfer computer program, shows a five percent or less difference in the calculated temperatures. Approximately ninety seconds of machine time is required to calculate the radial temperatures using (50) for paired values of β^* and t^* and two values of θ.

IV. CONCLUSION

The analytical expression of the transient temperature field in an isotropic, homogeneous, finite length, solid cylinder whose lateral surface is subjected to the heating by a nuclear thermal radiation environment has been derived. This expression provides a convenient means for the nondimensional representation and parametric analysis of the temperature field in the cylinder. In addition, this temperature equation can be used in the analysis of those effects dependent on temperature or temperature change, e.g., thermal stress in a cylinder.

---

[2] *J.D. Gaski, Chrysler Improved Numerical Differencing Analyzer for 3rd Generation Computers, TN-AP-67-287, October 20, 1967, Chrysler Corporation Space Division, New Orleans, Louisiana.*

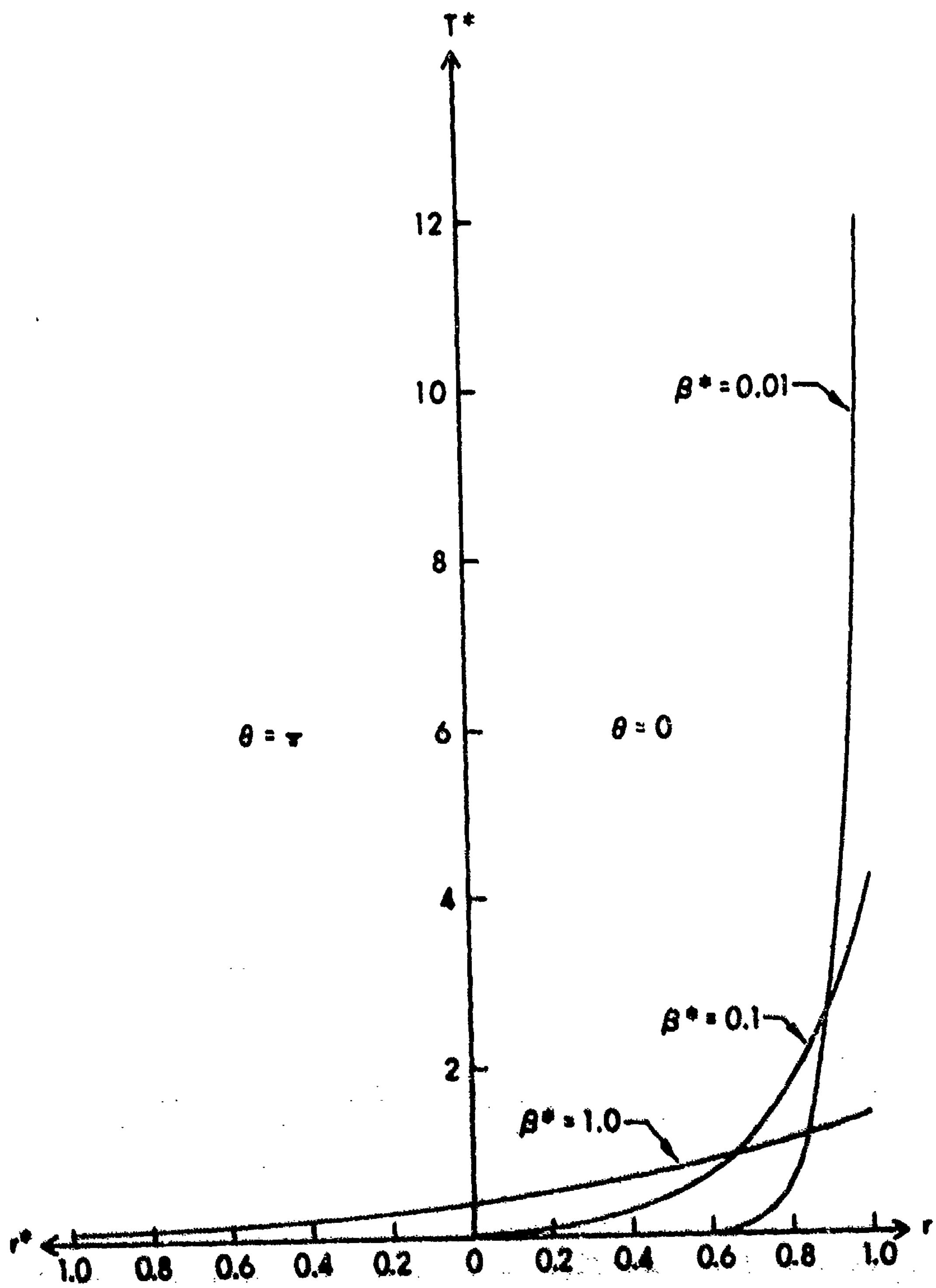

Figure 2. Radial Temperature Distribution on $\theta = 0$ and π for $t^* = 1.0$ and $\beta^* = 0.01$, 0.1, and 1.0.

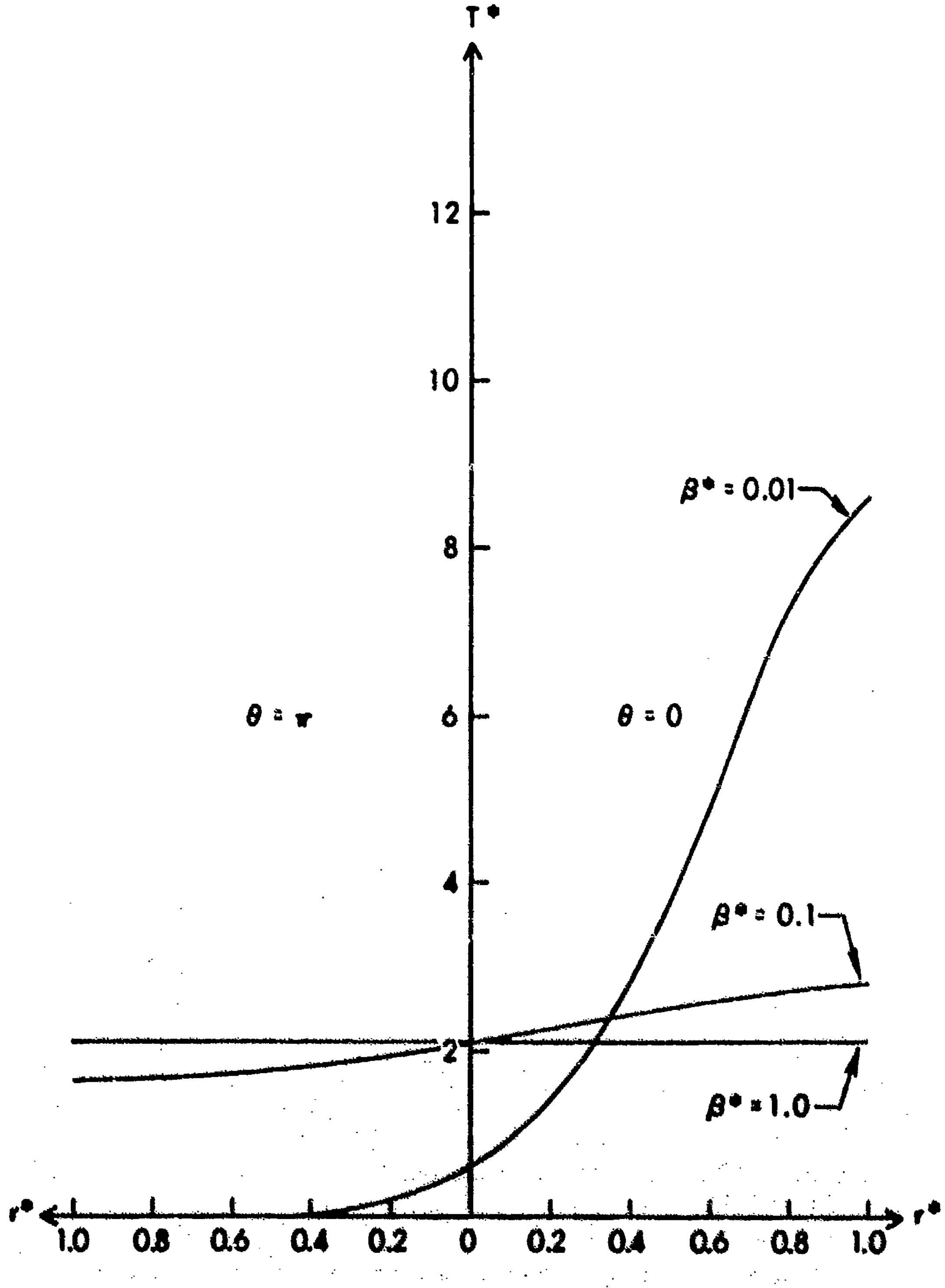

Figure 3. Radial Temperature Distribution on $\theta = 0$ and π for $t^*=10.0$ and $\beta^*=0.01$, 0.1, and 1.0.

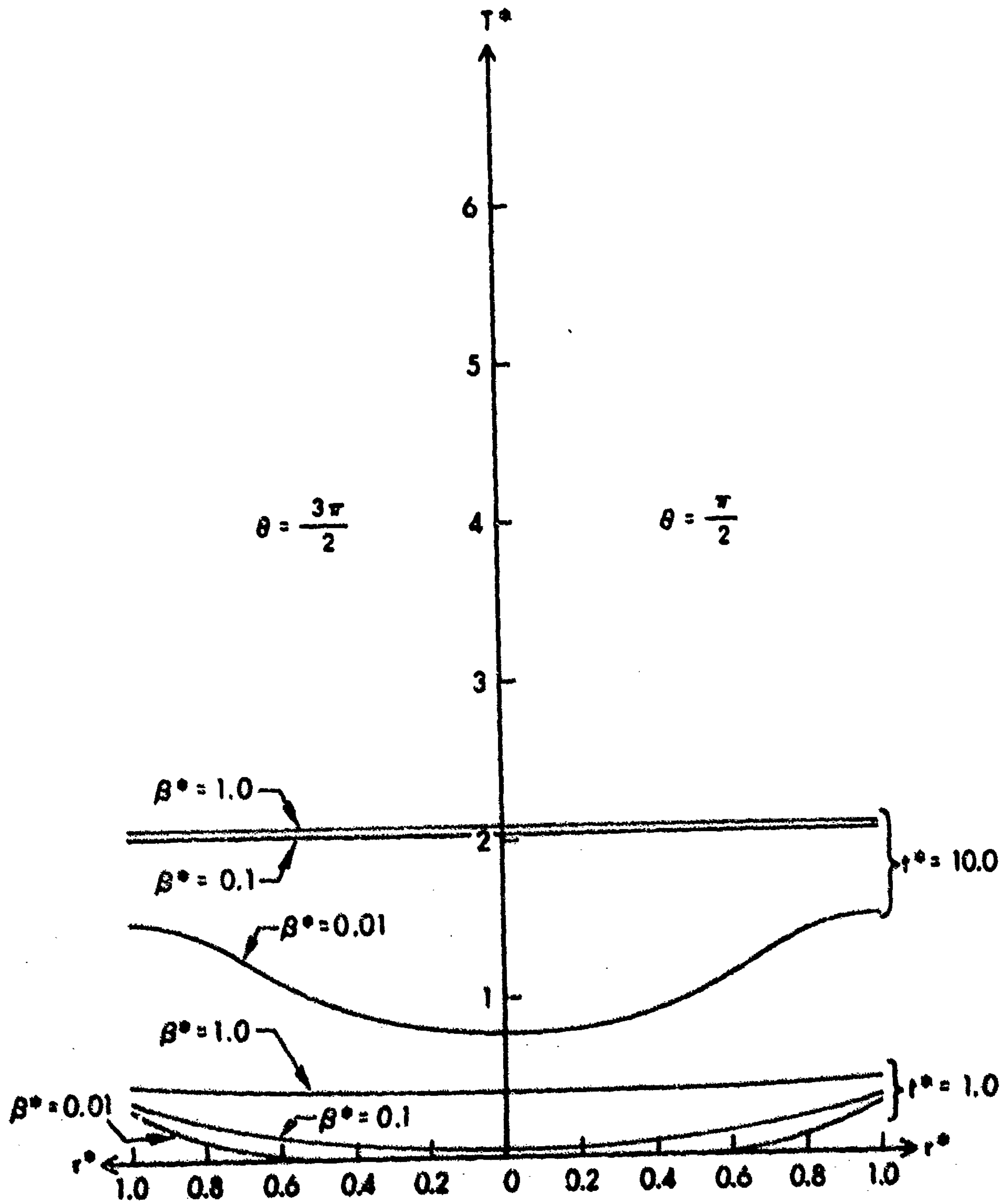

Figure 4. Radial Temperature Distribution on $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ for $t^* = 1.0$ and 10.0 and $\beta^* = 0.01$, 0.1, and 1.0

REFERENCES

1. J.U. Ojalvo, "Conduction with Time-Dependent Heat Sources and Boundary Conditions", International Journal of Heat and Mass Transfer, Volume 5, 1962, pp. 1105-1109.

2. J.D. Gaski, Chrysler Improved Numerical Differencing Analyzer for 3rd Generation Computers, TN-AP-67-287, October 20, 1967, Chrysler Corporation Space Division, New Orleans, Louisiana.

GLOSSARY OF TERMS

a_{mn}	=	Coefficients defined by (38) and (39)
b_{mn}	=	Coefficients defined by (44)
c	=	Specific heat
$f, f(t)$	=	Time dependent portion of (1)
h	=	Half length of cylinder
r, θ, z	=	Cylindrical coordinates
r_o	=	Radius of cylinder
r^*, t^*, T^*	=	Dimensionless variables defined by (51)
t	=	Time
t_o	=	Rise time of nuclear thermal pulse
A_m, A_{mn}, B_m, B_{mn}, C_m, D_m	=	Unknown coefficients in (24)
C_1	=	Coefficient defined by
C_m	=	Coefficient defined by
$H, H(t)$	=	Time dependent irradiance of nuclear thermal value
H_o	=	Maximum irradiance of nuclear thermal pulse
$J_m(x)$, $Y_m(x)$	=	Ordinary Bessel function of argument x
$J_m^1(x)$	=	$\dfrac{dJ_m(x)}{dx}$
$T, T(r, \theta, t)$	=	Temperature in cylinder
$T_o, T_o(r, \theta)$	=	Part of solution to
β^*	=	Dimensionless variable defined by
κ	=	Thermal conductivity
λ_{mn}	=	Separation constant of (13)
ρ	=	Density
Φ_{mn}	=	Part of solution to (3)
Ψ_{mn}	=	Part of solution to (3)

DISTRIBUTION LIST

No. of Copies	Organization
12	Commander Defense Documentation Center ATTN: DDC-TCA Cameron Station Alexandria, VA 22314
1	Commander US Army Materiel Development and Readiness Command ATTN: DRCDMA-ST 5001 Eisenhower Avenue Alexandria, VA 22333
1	Commander US Army Aviation Systems Command ATTN: DRSAV-E 12th and Spruce Streets St. Louis, MO 63166
1	Director US Army Air Mobility Research and Development Laboratory Ames Research Center Moffett Field, CA 94035
1	Commander US Army Electronics Command ATTN: DRSEL-RD Fort Monmouth, NJ 07703
1	Commander US Army Missile Command ATTN: DRSMI-R Redstone Arsenal, AL 35809
1	Commander US Army Tank Automotive Development Command ATTN: DRDTA-RWL Warren, MI 48090
2	Commander US Army Mobility Equipment Research and Development Command ATTN: Tech Docu Cen, Bldg 315 DRSME-RZT Fort Belvoir, VA 22060
1	Commander US Army Armament Command Rock Island, IL 61202
1	Commander US Army Harry Diamond Labs ATTN: DRXDO-NP 2800 Powder Mill Road Adelphi, MD 20783
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SA White Sands Missile Range NM 88002
1	Commander US Army Nuclear Agency ATTN: ACTN-W Fort Bliss, TX 79916
1	Commander Naval Surface Weapons Center ATTN: WR-42 (Mr. N. Griff) White Oak, MD 20910
1	AFWL-SAT (Mr. A. Sharp) Kirtland AFB, NM 87117

Aberdeen Proving Ground

Dir, USAMSAA
Marine Corps Ln Ofc